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NONISOTHERMIC FLOW OF GAS MIXTURE IN A CHANNEL AT INTERMEDIATE KNUDSEN NUMBERS*

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Solution of the problem of gas mixture flow in a plane channel at intermediate Knudsen numbers is considered on the basis of the 20-moment approximation as a function of distribution. The applied method consists of averaging moment equations valid throughout the flow region (including the Knudsen layers) with the determination of boundary values of macroscopic parameters on the wall using the approximate Loyalka method /1,2/. Expressions are obtained for a binary mixture for the mean molar velocity averaged over the channel cross section, difference of component velocities, and the relative heat flux in the presence of longitudinal gradients of partial pressures, and for the temperature gradients. Respective kinetic coefficients of the Onsager matrix are calculated. Dependence of these coefficients on the Knudsen number, and the properties of molecule scatter on the channel wall are analyzed in detail in the case of one-component gas and of a binary mixture with small relative difference of mass and diameters of molecule scatter.

1. Consider a slow flow of gas mixture in a plane channel bounded at $x = \pm d/2$ by two infinite parallel planes, assuming the existence in the z-direction of small relative gradients of partial pressure $k_\alpha = p_{\alpha 0}^{-1} dp_\alpha/dz$ and temperature $\tau = T_0^{-1} dT/dz$. It is possible to seek a solution of the distribution function of the form

$$f_\alpha(\mathbf{v}_\alpha, x, z) = f_{\alpha 0}\left[1 + k_\alpha z + \tau z\left(\beta_\alpha v_\alpha^2 - \frac{5}{2}\right) + \Phi_\alpha(\mathbf{v}_\alpha, x)\right] \quad (1.1)$$

$$f_{\alpha 0} = n_{\alpha 0}(m_\alpha/2\pi kT_0)^{3/2}\exp(-\beta_\alpha v_\alpha^2), \quad n_{\alpha 0} = \frac{p_{\alpha 0}}{kT_0}, \quad \beta_\alpha = \frac{m_\alpha}{2kT_0}$$

where subscript 0 corresponds to parameters of the Maxwell absolute distribution, and Φ_α an unbalanced addition to the distribution function, determined by the linearized kinetic Boltzmann equation /3/

$$v_{\alpha x}\frac{\partial \Phi_\alpha}{\partial x} + v_{\alpha z}k_\alpha + v_{\alpha z}\tau\left(\beta_\alpha v_\alpha^2 - \frac{5}{2}\right) = \sum_\beta L_{\alpha\beta}\Phi_\alpha \quad (1.2)$$

$$L_{\alpha\beta}\Phi_\alpha = \int f_{\beta 0}(\Phi_\alpha' + \Phi_\beta' - \Phi_\alpha - \Phi_\beta)\,|\mathbf{v}_\alpha - \mathbf{v}_\beta|\, b\, db\, d\varepsilon\, d\mathbf{v}_\beta$$

where the prime relates to velocities of molecules after collision.

The equations for moments of distribution function, which follow from (1.2) are used below. We restrict our considerations to the set of moment equations that on passing to the limit of continuous medium (the region of flow away from walls) correspond to the 20-moment Grad's approximation /4/. In connection with this the right-hand side of equations is presented in the form similar to the right-hand sides of moments obtaining by Grad's method /5/. Actually this form of presentation is based on the equivalence of moments of the collision integral and of the exact $L_{\alpha\beta}\Phi_\alpha$ and model $L_{\alpha\beta}^{(N)}\Phi_\alpha$ representations within the first N moment equations /6–8/. Since the definition at the level of moment equations proves to be practically adequate for obtaining the required results, this method corresponds as regards accuracy to the use of the true linearized Boltzmann operator of collisions.

Multiplying successively (1.2) by $\psi_\alpha(c_\alpha)\exp(-c_\alpha^2)$, where $\psi_\alpha = c_{\alpha i}$, $c_{\alpha i}c_{\alpha j} - {}^1/_3 c_\alpha^2\delta_{ij}$, $c_{\alpha i}(c_\alpha^2 - {}^5/_2)$ and $c_{\alpha i}c_{\alpha j}c_{\alpha k} - {}^1/_5 c_\alpha^2(c_{\alpha i}\delta_{jk} + c_{\alpha j}\delta_{ik} + c_{\alpha k}\delta_{ij})$, and $\mathbf{c}_\alpha = \beta_\alpha^{1/2}\mathbf{v}_\alpha$, and integrating with respect to velocities, we obtain for the plane geometric problem considered here moment equations of the form

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$$\frac{\partial}{\partial x} p_{\alpha xz} + p_{\alpha 0} k_\alpha = -\sum_\beta \left[\frac{kT_0 n_{\alpha 0} n_{\beta 0}}{n_0 [D_{\alpha\beta}]_1} (u_{\alpha z} - u_{\beta z}) + \xi_{\alpha\beta} \left[\frac{h_{\alpha z}}{m_\alpha n_{\alpha 0}} - \frac{h_{\beta z}}{m_\beta n_{\beta 0}} \right] \right. \tag{1.3}$$

$$\frac{\partial}{\partial x} \left(m_\alpha s_{\alpha zxx} + \frac{2}{5} h_{\alpha z} + p_{\alpha 0} u_{\alpha z} \right) = - p_0{}^2 \sum_\beta \frac{a_{\alpha\beta} p_{\beta xz}}{p_{\beta 0}} \tag{1.4}$$

$$\frac{2}{5} \frac{\partial}{\partial x} \left(\Pi_{\alpha zxxx} + \Pi_{\alpha zxyy} + \Pi_{\alpha zxzz} - \frac{5}{2} p_{\alpha xz} \right) + p_{\alpha 0} \tau = - \frac{kT_0}{m_\alpha} \sum_\beta \xi_{\alpha\beta} (u_{\alpha z} - u_{\beta z}) - \frac{p_0^2}{T_0} \sum_\beta \frac{b_{\alpha\beta} h_{\beta z}}{p_{\beta 0}} \tag{1.5}$$

$$\frac{\partial}{\partial x} (4\Pi_{\alpha zxxx} - \Pi_{\alpha zxyy} - \Pi_{\alpha zxzz}) = - \frac{25}{4} \frac{p_0^2}{T_0} \sum_\beta \frac{d_{\alpha\beta} m_\beta s_{\beta zxx}}{p_{\beta 0}} \tag{1.6}$$

$$\frac{\partial}{\partial x} (4\Pi_{\alpha zxyy} - \Pi_{\alpha zxxx} - \Pi_{\alpha zxzz}) = - \frac{25}{4} \frac{p_0^2}{T_0} \sum_\beta \frac{d_{\alpha\beta} m_\beta s_{\beta zyy}}{p_{\beta 0}} \tag{1.7}$$

$$s_{\alpha zxx} + s_{\alpha zyy} + s_{\alpha zzz} = 0 \tag{1.8}$$

where $u_{\alpha z}$ is the mean velocity of the mixture component α, $p_{\alpha xz}$ is the partial tensor of viscous stresses, and $h_{\alpha z}$ is the partial relative heat flux. Their expressions and, also, those for moments of the third $s_{\alpha ijk}$ and fourth $\Pi_{\alpha ijkl}$ order are of the form

$$\left\| \begin{matrix} u_{\alpha i} \\ p_{\alpha ij} \\ h_{\alpha i} \\ m_\alpha s_{\alpha ijk} \\ \Pi_{\alpha ijkl} \end{matrix} \right\| = 2 p_{\alpha 0} \beta_\alpha^{-1/2} \pi^{-3/2} \int \left\| \begin{matrix} (2p_{\alpha 0})^{-1} c_{\alpha i} \\ \beta_\alpha^{1/2} \left(c_{\alpha i} c_{\alpha j} - \frac{1}{3} c_\alpha{}^2 \delta_{ij} \right) \\ \frac{1}{2} c_{\alpha i} \left(c_\alpha{}^2 - \frac{5}{2} \right) \\ c_{\alpha i} c_{\alpha j} c_{\alpha k} - \frac{1}{5} c_\alpha{}^2 (c_{\alpha i} \delta_{jk} + \\ c_{\alpha j} \delta_{ik} + c_{\alpha k} \delta_{ij}) \\ \beta_\alpha^{1/2} c_{\alpha i} c_{\alpha j} c_{\alpha k} c_{\alpha l} \end{matrix} \right\| \Phi_\alpha \exp(-c_\alpha{}^2) \, dc_\alpha \tag{1.9}$$

In Eqs. (1.3)—(1.7) $[D_{\alpha\beta}]_1 = 3kT_0/(16 n_0 \mu_{\alpha\beta} \Omega_{\alpha\beta}{}^{11})$ corresponds to the first approximation of the coefficient of interdiffusion of the binary mixture of α and β molecules /9/, and coefficients $\xi_{\alpha\beta}$, $a_{\alpha\beta}$, $b_{\alpha\beta}$ were determined in /5/, where the moment equation was given in the approximation of 13 moments. For new coefficients appearing at passing to the 20 moment approximation respective calculations yield

$$d_{\alpha\alpha} = \frac{9}{4} \frac{y_\alpha{}^2}{[\lambda_{\alpha\alpha}]_1} + \frac{4}{25} \frac{T_0}{p_0} \sum_{\beta \neq \alpha} \frac{y_\alpha y_\beta}{(m_\alpha + m_\beta)^2 [D_{\alpha\beta}]_1} \left(\frac{15}{2} m_\alpha{}^2 + 9 m_\alpha m_\beta A^*_{\alpha\beta} + \frac{24}{7} m_\beta{}^2 D_{\alpha\beta}{}^* \right)$$

$$d_{\alpha\beta} = - \frac{4}{25} \frac{T_0}{p_0} \frac{m_\alpha m_\beta y_\alpha y_\beta}{(m_\alpha + m_\beta)^2 [D_{\alpha\beta}]_1} \left(\frac{15}{2} - 9 A^*_{\alpha\beta} + \frac{24}{7} D^*_{\alpha\beta} \right), \ \beta \neq \alpha$$

$$y_\alpha = \frac{n_\alpha}{n}, \quad A^*_{\alpha\beta} = \frac{\Omega^{22}_{\alpha\beta}}{2\Omega^{11}_{\alpha\beta}}, \quad D^*_{\alpha\beta} = \frac{5\Omega^{33}_{\alpha\beta} - 3\Omega^{13}_{\alpha\beta}}{24\Omega^{11}_{\alpha\beta}}$$

where y_α is the relative molar concentration of the mixture α component and $\Omega_{\alpha\beta}{}^{lq}$ corresponds to the Chapman—Cowling integrals /9/.

Away from the walls the system of equations must correspond to the usual 20 moment Grad's approximation /4/. For the unbalanced addition to the distribution function in this region we obtain, with allowance for the smallness of quantities $u^a_{\alpha z}$, $p^a_{\alpha xz}$, $h^a_{\alpha z}$, $s^a_{\alpha ijk}$ the formula

$$\Phi_\alpha{}^a (c_\alpha, x) = 2\beta_\alpha^{1/2} (u_z{}^a + w^a_{\alpha z}) c_{\alpha z} + 2 p_{\alpha 0}^{-1} p^a_{\alpha xz} c_{\alpha x} c_{\alpha z} + \tag{1.10}$$
$$\frac{4}{5} \beta_\alpha^{1/2} p_{\alpha 0}^{-1} h^a_{\alpha z} c_{\alpha z} \left(c_\alpha{}^2 - \frac{5}{2} \right) + 2\beta_\alpha^{1/2} p_{\alpha 0}^{-1} m_\alpha (s^a_{\alpha zxx} c^2_{\alpha x} + s^a_{\alpha zyy} c^2_{\alpha y} + {}^1\!/_3 s^a_{\alpha zzz} c^2_{\alpha z}), \quad w^a_{\alpha z} = u^a_{\alpha z} - u_z{}^a$$

where u_z is the mean-mass velocity of mixture and superscripts a denotes asymptotic values of respective quantities, i.e. values outside the Knudsen layer.

Substituting (1.10) into $\Pi_{\alpha ijkl}$ of (1.9) and integrating with respect to velocities we obtain

$$\Pi^a_{\alpha zxxx} = {}^3/_2 p^a_{\alpha xz}, \quad \Pi^a_{\alpha zxyy} = {}^1/_2 p^a_{\alpha xz}, \quad \Pi^a_{\alpha zxzz} = {}^3/_2 p^a_{\alpha xz} \tag{1.11}$$

Solution of the system of Eqs.(1.3)—(1.8) with allowance for (1.11) and the problem symmetry relative to the longitudinal axis of the channel ($\Phi_\alpha(x, c_{\alpha x}, c_{\alpha y}, c_{\alpha z}) = \Phi_\alpha(-x, -c_{\alpha x}, c_{\alpha y}, c_{\alpha z})$) enables us to obtain explicit expressions for $w^a_{\alpha z}, p^a_{\alpha xz}, h^a_{\alpha z}, s^a_{\alpha ijk}$.

In the case of a binary mixture ($\alpha, \beta = 1,2$) the respective solutions are of the form

$$w^a_{\alpha z} = (-1)^\alpha \frac{[D_{\alpha\beta}]_2 \rho_{\beta 0}}{\rho_0 y_\alpha y_\beta}\Big(\frac{d}{dz} y_1 + y_\alpha y_\beta [\alpha_p]_2 p_0^{-1}\frac{dp}{dz} + \tag{1.12}$$

$$y_\alpha y_\beta [\alpha_T]_1 T_0^{-1}\frac{dT}{dz}\Big), \quad \beta \neq \alpha; \quad p^a_{\alpha xz}(x) = -x\frac{\eta_\alpha}{\eta}\frac{dp}{dz}$$

$$h^a_{\alpha z} = -\frac{5}{2}\frac{a_\alpha p_0}{y_\beta}[D_{\alpha\beta}]_2 \frac{d}{dz} y_1 + \frac{2T_0 y_\alpha}{5p_0|b|}\alpha'_{p\alpha}\frac{dp}{dz} - \lambda'_\alpha \frac{dT}{dz}, \quad \beta \neq \alpha$$

$$s^a_{\alpha zxx} = \frac{16 y_\alpha T_0}{25 p_0 m_\alpha}\delta_\alpha \frac{dp}{dz}, \quad s^a_{\alpha zyy} = -\frac{1}{4}s^a_{\alpha zxx}, \quad s^a_{\alpha zzz} = -\frac{3}{4}s^a_{\alpha zxx}$$

$$\rho_0 = \sum_\alpha \rho_{\alpha 0}, \quad \rho_{\alpha 0} = m_\alpha n_{\alpha 0}, \quad |b| = b_{11}b_{22} - b_{12}b_{21}$$

$$\lambda_1' = \lambda_1 + {}^5/_2 a_1 y_1 n_0 k [D_{12}]_2 [\alpha_T]_1$$

$$\lambda_2' = \lambda_2 + {}^5/_2 a_2 y_2 n_0 k [D_{12}]_2 [\alpha_T]_1$$

$$\alpha'_{p1} = (b_{22}\eta_1 - b_{12}\eta_2)\eta^{-1} - \frac{25}{4} a_1 n_0 k |b| [D_{12}]_2 [\alpha_p]_2$$

$$\alpha'_{p2} = (b_{11}\eta_2 - b_{21}\eta_1)\eta^{-1} - \frac{25}{4} a_2 n_0 k |b| [D_{12}]_2 [\alpha_p]_2$$

$$\delta_1 = (d_{22}\eta_1 - d_{21}\eta_2)(\eta\,|d|)^{-1}, \quad \delta_2 = (d_{11}\eta_2 - d_{12}\eta_1)(\eta\,|d|)^{-1}$$

$$a_1 = -\frac{2\xi_{12}}{5n_0^2 k |b|}\Big(\frac{b_{22}}{m_1} + \frac{b_{21}}{m_2}\Big), \quad a_2 = \frac{2\xi_{12}}{5n_0^2 k|b|}\Big(\frac{b_{11}}{m_2} + \frac{b_{12}}{m_1}\Big)$$

$$\xi_{12} = \frac{n_{10}n_{20}}{n_0 [D_{12}]_1}\mu_{12}\Big(\frac{6}{5}C_{12}^* - 1\Big), \quad C_{12}^* = \frac{\Omega_{12}^{12}}{3\Omega_{12}^{11}}, \quad \mu_{12} = \frac{m_1 m_2}{m_1 + m_2}$$

Expressions for partial coefficients of viscosity η_α and thermal conductivity λ_α and, also, for the second order interdiffusion coefficient $[D_{12}]_2$, and for the baro- and thermodiffusion constants $[\alpha_p]_2$ and $[\alpha_T]_1$ are given in /5/.

2. Let us now obtain the averaged over the channel cross section expressions for the relative heat flux, the difference of velocities of mixture components, and for the mean molar velocity of the mixture.

Summing (1.3) over α and integrating with respect to x, we obtain for $p_{xz}(x)$

$$p_{xz}(x) = -x\frac{dp}{dz}; \quad p_{xz} = \sum_\alpha p_{\alpha xz}, \quad p = \sum_\alpha p_\alpha \tag{2.1}$$

that is valid throughout the flow region.

Expressions for $p_{\alpha xz}(x)$ are obtained from the solution of Eqs.(1.4). The substitution of these expressions into (2.1) and integration of the obtained relation with respect to x yields

$$p_0^{-1}\sum_\alpha \eta_\alpha y_\alpha^{-1}\Big\{m_\alpha\Big[s_{\alpha zxx}(x) - s_{\alpha zxx}\Big(\frac{d}{2}\Big)\Big] + \frac{2}{5}\Big[h_{\alpha z}(x) - h_{\alpha z}\Big(\frac{d}{2}\Big)\Big] + \tag{2.2}$$

$$p_{\alpha 0}\Big[u_{\alpha z}(x) - u_{\alpha z}\Big(\frac{d}{2}\Big)\Big]\Big\} = \frac{1}{2}\Big(x^2 - \frac{d^2}{4}\Big)\frac{dp}{dz}$$

We average Eqs.(1.3), (1.5), (1.6), and (2.2) over the channel cross section. As the result we have

$$-\sum_\beta [y_\alpha y_\beta p_0 [D_{\alpha\beta}]_1^{-1}(\langle u_{\alpha z}\rangle - \langle u_{\beta z}\rangle) + \xi_{\alpha\beta}\Big(\frac{\langle h_{\alpha z}\rangle}{m_\alpha n_{\alpha 0}} - \frac{\langle h_{\beta z}\rangle}{m_\beta n_{\beta 0}}\Big)\Big] = K_{\alpha 1} \tag{2.3}$$

$$-\frac{kT_0}{m_\alpha}\sum_\beta \xi_{\alpha\beta}(\langle u_{\alpha z}\rangle - \langle u_{\beta z}\rangle) - \frac{p_0^2}{T_0}\sum_\beta \frac{b_{\alpha\beta}\langle h_{\beta z}\rangle}{p_{\beta 0}} = K_{\alpha 2} \tag{2.4}$$

$$-\frac{5}{4}\frac{p_0^2}{T_0}\sum_\beta \frac{d_{\alpha\beta} m_\beta \langle s_{\beta zxx}\rangle}{p_{\beta 0}} = K_{\alpha 3} \tag{2.5}$$

$$p_0^{-1}\sum_\beta \eta_\beta y_\beta^{-1}\left(m_\beta\langle s_{\beta zxx}\rangle+\frac{2}{5}\langle h_{\beta z}\rangle+p_{\beta0}\langle u_{\beta z}\rangle\right)=K_4 \tag{2.6}$$

$$K_{\alpha1}=\frac{2}{d}p_{\alpha xz}\left(\frac{d}{2}\right)+p_{\alpha0}k_\alpha,\quad K_{\alpha2}=\frac{4}{5d}\left[\Pi_{\alpha zxxx}\left(\frac{d}{2}\right)+\Pi_{\alpha zxyy}\left(\frac{d}{2}\right)+\Pi_{\alpha zxzz}\left(\frac{d}{2}\right)-\frac{5}{2}p_{\alpha xz}\left(\frac{d}{2}\right)\right]+p_{\alpha0}\tau$$

$$K_{\alpha3}=\frac{2}{5d}\left[4\Pi_{\alpha zxxx}\left(\frac{d}{2}\right)-\Pi_{\alpha zxyy}\left(\frac{d}{2}\right)-\Pi_{\alpha zxzz}\left(\frac{d}{2}\right)\right]$$

$$K_4=p_0^{-1}\sum_\beta \eta_\beta y_\beta^{-1}\left[m_\beta s_{\beta zxx}\left(\frac{d}{2}\right)+\frac{2}{5}h_{\beta z}\left(\frac{d}{2}\right)+p_{\beta0}u_{\beta z}\left(\frac{d}{2}\right)\right]-\frac{d^2}{12}\frac{dp}{dz}$$

moreover,

$$\langle Q_\alpha\rangle=\frac{1}{d}\int_{-d/2}^{d/2}Q_\alpha(x)\,dx$$

In the case of a binary mixture the solution of system (2.3)— (2.6) yields

$$\langle h_z\rangle=\langle h_{1z}\rangle+\langle h_{2z}\rangle=-[K_{11}[D_{12}]_2[\alpha_T]_1+T_0p_0^{-1}(K_{12}\lambda_1'y_1^{-1}+K_{22}\lambda_2'y_2^{-1})] \tag{2.7}$$

$$\langle u_{1z}\rangle-\langle u_{2z}\rangle=-\frac{[D_{12}]_2}{p_0y_1y_2}\left[K_{11}+\frac{5}{2}(K_{12}a_1+K_{22}a_2)\right]$$

$$\langle u_{mz}\rangle=y_1\langle u_{1z}\rangle+y_2\langle u_{2z}\rangle=-K_{11}p_0^{-1}[D_{12}]_2[\alpha_p]_2+\frac{2T_0}{5p_0^2|b|}(K_{12}\alpha_{p1}'+K_{22}\alpha_{p2}')+\frac{4T_0}{5p_0^2}(K_{13}\delta_1+K_{23}\delta_2)+K_4\eta^{-1}$$

3. For the determination of unknown quantities on the channel wall we use the approximate method /1/. It was shown in /1,2/ that the application of that method corresponds to the simplest choice of the trial function within the more general variational method developed in /10/.

We introduce the distribution function of incident and reflected molecules so that $\Phi_\alpha=\Phi_\alpha^+$ when $c_{\alpha x}>0$ and $\Phi_\alpha=\Phi_\alpha^-$ when $c_{\alpha x}<0$, $c_{\alpha x}>0$ correspond to the positive direction of the x axis. In conformity with (1.10) and with allowance for the usual Maxwell condition of molecule reflection on a wall, for function $\Phi_\alpha^\pm$ at $x=d/2$ we have

$$\Phi_\alpha^+(\mathbf{c}_\alpha,d/2)=2\beta_\alpha^{1/2}(a+w_{\alpha z}^a)c_{\alpha z}+2p_{\alpha0}^{-1}p_{\alpha xz}^a(d/2)c_{\alpha x}c_{\alpha z}+{}^4/_5\beta_\alpha^{1/2}p_{\alpha0}^{-1}h_{\alpha z}^ac_{\alpha z}(c_\alpha^2-{}^5/_2)+2\beta_\alpha^{1/2}p_{\alpha0}^{-1}m_\alpha(s_{\alpha zxx}^ac_{\alpha x}^2+s_{\alpha zyy}^ac_{\alpha y}^2+{}^1/_3s_{\alpha zzz}^ac_{\alpha z}^2) \tag{3.1}$$

$$\Phi_\alpha^-(\mathbf{c}_\alpha,d/2)=(1-\varkappa_\alpha)\Phi_\alpha^+(-c_{\alpha x},c_{\alpha y},c_{\alpha z},d/2)$$

where $\varkappa_\alpha$ is the fraction of molecules that have undergone diffusion reflection on the wall, and $u_z^a(d/2)$ is replaced by an arbitrary constant a.

Using condition (2.1) and the definition of $p_{\alpha xz}$ on the wall, after the calculation of respective integrals with (3.1) taken into account, we obtain the constant a expressed in terms of concentration, temperature, and pressure gradients

$$a=-\left(\sum_\alpha\varkappa_\alpha\beta_\alpha^{1/2}p_{\alpha0}\right)^{-1}\left[\frac{\sqrt{\pi}}{2}d\left(1-\frac{1}{2\eta}\sum_\alpha\varkappa_\alpha\eta_\alpha\right)\frac{dp}{dz}+\sum_\alpha\varkappa_\alpha\beta_\alpha^{1/2}\left(p_{\alpha0}w_{\alpha z}^a+\frac{1}{5}h_{\alpha z}^a+\frac{m_\alpha}{2}s_{\alpha zxx}^a\right)\right] \tag{3.2}$$

whose substitution into (3.1) yields the unknown quantities on the channel wall appearing in K_{ik}. As the result the considered here quantities of streams are expressed in the form of linear combinations of respective thermodynamic quantities.

According to the thermodynamics of irreversible processes for discontinuous systems /11/ the relation between streams and gradients can be represented thus

$$\left\|\begin{matrix}\langle h_z\rangle\\ \langle u_{1z}\rangle-\langle u_{2z}\rangle\\ \langle u_{mz}\rangle\end{matrix}\right\|=-\left\|\begin{matrix}\Lambda_{qq}&\Lambda_{q1}&\Lambda_{qm}\\ \Lambda_{1q}&\Lambda_{11}&\Lambda_{1m}\\ \Lambda_{mq}&\Lambda_{m1}&\Lambda_{mm}\end{matrix}\right\|\left\|\begin{matrix}T^{-2}\frac{dT}{dz}\\ pT^{-1}\frac{d}{dz}y_1\\ T^{-1}\frac{dp}{dz}\end{matrix}\right\| \tag{3.3}$$

The general expression for the kinetic coefficients Λ_{ik} obtained by comparing (3.3) and (2.7) are of the form

$$
\begin{aligned}
\Lambda_{11} &= \frac{[D_{12}]_2 T_0}{p_0 y_1 y_2}\left[1 - \frac{B[D_{12}]_2}{2\sqrt{\pi} d p_0 y_1 y_2}(T_1 T_2 t^2 + 12 T_1 a_1^2 + 12 T_2 a_2^2)\right] \\
\Lambda_{1m} &= \Lambda_{m1} = \frac{[D_{12}]_2 T_0}{p_0}\Big\{[\alpha_p]_2 + \frac{1}{4\eta y_1 y_2}[\eta_1(2-\varkappa_1)F_1 - \\
&\quad \eta_2(2-\varkappa_2)F_2]\Big\} - \frac{BT_1T_2}{\sqrt{\pi} p_0 d y_1 y_2}[D_{12}]_2[\alpha_p]_2 t + \\
&\quad \frac{2BT_0}{25\sqrt{\pi} d p_0^2 |b| y_1 y_2}(T_1 \alpha'_{p1} F_3 - T_2 \alpha'_{p2} F_4) + \\
&\quad \frac{8BT_0}{25\sqrt{\pi} d p_0^2 y_1 y_2}(\delta_1 T_1 F_5 - \delta_2 T_2 F_6) \\
\Lambda_{1q} &= \Lambda_{q1} = [D_{12}]_2 T_0\Big[[\alpha_T]_1 - \frac{BT_1T_2}{\sqrt{\pi} d p_0 y_1 y_2}[D_{12}]_2[\alpha_T]_1 t - \\
&\quad \frac{BT_0}{5\sqrt{\pi} d p_0^2 y_1^2 y_2^2}(T_1 y_2 \lambda_1' F_3 - T_2 y_1 \lambda_2' F_4)\Big]
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
\Lambda_{qq} &= T_0^2\lambda' - \frac{2BT_1T_2T_0}{\sqrt{\pi} d}[D_{12}]_2^2[\alpha_T]_1^2 - \\
&\quad \frac{4BT_1T_2T_0^2}{5\sqrt{\pi} d p_0 y_1 y_2}[D_{12}]_2[\alpha_T]_1(\lambda_1' y_2 - \lambda_2' y_1) - \\
&\quad \frac{2BT_0^3}{25\sqrt{\pi} d p_0^2 y_1^2 y_2^2}[(13 - T_1)T_1 y_2^2(\lambda_1')^2 - 2T_1T_2 y_1 y_2 \lambda_1'\lambda_2' + \\
&\quad (13 - T_2)T_2 y_1^2(\lambda_2')^2] \\
\Lambda_{mm} &= \frac{T_0 d^2}{12\eta} + \frac{T_0 d}{2p_0}\left[\frac{\sqrt{\pi}\varepsilon^2 p_0}{B} + \sum_\alpha \beta_\alpha^{-1/2}\frac{(2-\varkappa_\alpha)}{\sqrt{\pi} y_\alpha}\left(\frac{\eta_\alpha}{\eta}\right)^2\right] + \\
&\quad \frac{4T_0^2}{25p_0^2}\sum_\alpha\left\{\left(\frac{\alpha'_{p\alpha}}{|b|} + 4\delta_\alpha\right)\left[\varepsilon T_\alpha - (1-\varkappa_\alpha)\frac{\eta_\alpha}{\eta}\right]\right\} + \\
&\quad \frac{8BT_0^2T_1T_2}{25\sqrt{\pi} d p_0^3}[D_{12}]_2[\alpha_p]_2\Big[\frac{1}{|b|}(\alpha'_{p1} - \alpha'_{p2}) + 4(\delta_1 - \delta_2) - \\
&\quad \frac{25p_0}{4T_0}[D_{12}]_2[\alpha_p]_2\Big] + \frac{T_0}{p_0}[D_{12}]_2[\alpha_p]_2\Big[y_1 - 2\varepsilon T_1 + \\
&\quad \frac{\eta_1}{\eta}(1-\varkappa_1)\Big] - \frac{8T_0^3 B}{625\sqrt{\pi} d p_0^4}\sum_\alpha\Big\{T_\alpha\Big[(13 - T_\alpha)\left(\frac{\alpha'_{p\alpha}}{|b|}\right)^2 + \\
&\quad 8\frac{\alpha'_{p\alpha}}{|b|}\delta_\alpha(3 - T_\alpha) + 8\delta_\alpha^2(11 - 2T_\alpha)\Big]\Big\} + \\
&\quad \frac{16T_0^3BT_1T_2}{625\sqrt{\pi} d p_0^4}\left[\frac{\alpha'_{p1}\alpha'_{p2}}{|b|^2} + \frac{4}{|b|}(\alpha'_{p1}\delta_2 + \alpha'_{p2}\delta_1) + 16\delta_1\delta_2\right] \\
\Lambda_{mq} &= \Lambda_{qm} = [D_{12}]_2[\alpha_T]_1 T_0\left(y_1 - T_1\varepsilon - \frac{\varkappa_1\eta_1}{2\eta}\right) - \\
&\quad \frac{T_0^2}{5p_0}\sum_\alpha\left[\lambda_\alpha' y_\alpha^{-1}\left(T_\alpha\varepsilon + \frac{\varkappa_\alpha\eta_\alpha}{\eta}\right)\right] + \frac{4T_0^3B}{125\sqrt{\pi} p_0^2 d}\Big\{\frac{\lambda_1' T_1}{y_1 p_0}\Big[(13 - \\
&\quad T_1)\frac{\alpha'_{p1}}{|b|} - T_2\frac{\alpha'_{p2}}{|b|} + 4\delta_1(3 - T_1) - 4T_2\delta_2 - \frac{25T_2p_0}{2T_0}[D_{12}]_2[\alpha_p]_2\Big] + \\
&\quad \frac{\lambda_2' T_2}{y_2 p_0}\Big[(13 - T_2)\frac{\alpha'_{p2}}{|b|} - T_1\frac{\alpha'_{p1}}{|b|} + 4\delta_2(3 - T_2) - 4T_1\delta_1 + \\
&\quad \frac{25T_1p_0}{2T_0}[D_{12}]_2[\alpha_p]_2\Big] + \frac{5T_1T_2}{T_0}[D_{12}]_2[\alpha_T]_1\Big[\frac{1}{|b|}(\alpha'_{p1} - \alpha'_{p2}) + \\
&\quad 4(\delta_1 - \delta_2) - \frac{25p_0}{2T_0}[D_{12}]_2[\alpha_p]_2\Big]\Big\}
\end{aligned} \tag{3.5}
$$

$F_1 = T_2 t + a_1$, $F_2 = T_1 t - a_2$, $F_3 = T_2 t + 12a_1$, $F_4 = T_1 t - 12a_2$, $F_5 = T_2 t + 2a_1$, $F_6 = T_1 t - 2a_2$, $t = 2 + a_1 - a_2$,

$\lambda' = \lambda_1' + \lambda_2'$, $\varepsilon = 1 - (\varkappa_1\eta_1 + \varkappa_2\eta_2)(2\eta)^{-1}$

$B = \sum_\alpha \beta_\alpha^{1/2}\varkappa_\alpha p_{\alpha 0}$, $\quad T_\alpha = \beta_\alpha^{1/2}\varkappa_\alpha p_{\alpha 0} B^{-1}$; $\quad \alpha = 1, 2$

4. Consider the case of the one-component gas $(y_1 = 1)$

$$\langle h_z \rangle = -\Lambda_{qq} T^{-2} \frac{dT}{dz} - \Lambda_{qm} T^{-1} \frac{dp}{dz}$$
$$\langle u_z \rangle = -\Lambda_{mq} T^{-2} \frac{dT}{dz} - \Lambda_{mm} T^{-1} \frac{dp}{dz}$$

Using (3.5) we obtain

$$\Lambda_{mm} = \beta^{-1/2} \frac{dT_0}{p_0} \left[\frac{\mathrm{Kn}^{-1}}{12} + \frac{(2-\varkappa)}{2} \left(\frac{1}{\sqrt{\pi}} + \frac{2-\varkappa}{\varkappa} \frac{\sqrt{\pi}}{4} \right) + \frac{5\varkappa \mathrm{Kn}}{12} - \frac{3\varkappa \mathrm{Kn}^2}{2\sqrt{\pi}} \right]$$
$$\Lambda_{qm} = \Lambda_{mq} = \beta^{-1/2} dT_0 \left[-\frac{3}{16}(2+\varkappa)\mathrm{Kn} + \frac{7\varkappa}{4\sqrt{\pi}} \mathrm{Kn}^2 \right]$$
$$\Lambda_{qq} = \beta^{-1/2} dT_0 p_0 \left(\frac{15}{8}\mathrm{Kn} - \frac{27\varkappa}{8\sqrt{\pi}} \mathrm{Kn}^2 \right), \quad \mathrm{Kn} = \frac{[\eta_1]}{p_0 \beta^{1/2} d}$$

where $[\eta]_1$ is viscosity coefficient that corresponds to the first approximation of the expansion in Sonin polynomials in the Chapman–Enskog method /9/.

We introduce the dimensionless quantities

$$J_m{}^* = J_m/(mJ_0) = 2\beta^{1/2} \langle u_z \rangle, \quad J_q{}^* = J_q/(kT_0 J_0) = 2\beta^{1/2} p_0^{-1} \langle h_z \rangle$$

where J_m and J_q are the respective averaged mass and heat fluxes per unit of channel cross section, and $J_0 = n_0/(2\beta^{1/2})$, moreover

$$J_m{}^* = -L_{mm} kd - L_{mq} \tau d, \quad J_q{}^* = -L_{qm} kd - L_{qq} \tau d$$

In the case of fully diffused reflection $(\varkappa = 1)$ these coefficients assume the form

$$L_{mm} = \frac{1}{6}\mathrm{Kn}^{-1} + \sigma + \frac{5}{6}\mathrm{Kn} - \frac{3}{\sqrt{\pi}}\mathrm{Kn}^2$$
$$L_{mq} = L_{qm} = -a_T \mathrm{Kn} + \frac{7}{2\sqrt{\pi}}\mathrm{Kn}^2, \quad L_{qq} = \frac{15}{4}\mathrm{Kn} - \frac{27}{4\sqrt{\pi}}\mathrm{Kn}^2$$

where $\sigma = 1/\sqrt{\pi} + \sqrt{\pi}/4 = 1.0073$ and $a_T = 9/8$ are the viscous and thermal slip coefficients whose values coincide with those obtained by the variational method in /10/.

For comparison we adduce the respective values of kinetic coefficients obtained in /12/, where second order slip effects (which corresponds to using the Barnett term in the expansion of the distribution function) were taken into account. When $\varkappa = 1$ we have

$$L_{mm} = \frac{1}{6}\mathrm{Kn}^{-1} + \sigma + \frac{3}{2}\mathrm{Kn} - \frac{9}{\sqrt{\pi}}\mathrm{Kn}^2$$
$$L_{mq} = L_{qm} = -a_T \mathrm{Kn} + \frac{9}{2\sqrt{\pi}}\mathrm{Kn}^2, \quad L_{qq} = \frac{15}{4}\mathrm{Kn} - \frac{27}{4\sqrt{\pi}}\mathrm{Kn}^2$$

It will be seen that the discrepancy in coefficient L_{mm}, which defines the isothermal Poiseuille transport of gas in the channel, begins with the term $\sim \mathrm{Kn}$ or $\sim \mathrm{Kn}^2$ relative to the conventional Poiseuille term of order Kn^{-1}. In cross coefficients the discrepancy appears already in the term of order Kn relative to the term that defines thermal slip of gas in the channel. Analysis shows that this discrepancy is entirely due to the incorrect choice in /12/ of the Barnett term structure taken there by simple analogy with its form in the particular case of the BGK model (by substituting 2/3 for 1 for the Prandtl number). However, it follows, for instance from /13/ that even for Maxwellian molecules the form of that term is more complex. In the case of arbitrary law of interaction the respective structure of the Barnett term is revealed by expansion (1.10) with the form of $h_{\alpha z}^a$ and $s_{\alpha ijk}^a$ taken into account for a one-component gas.

Owing to the unwieldiness of expression for Λ_{ik} in the case of a mixture, we shall analyze only those coefficients that appear in the expression for the diffusion flux (or the difference of component velocities averaged over the cross section) in the channel, since it is these coefficients (and the equal to them cross coefficients in expressions for $\langle h_z \rangle$ and $\langle u_{mz} \rangle$) that basically define the specific properties of phenomenon arising in investigations of gas mixtures.

Let us represent $\langle u_{1z} \rangle - \langle u_{2z} \rangle$ in the form

$$\langle u_{1z}\rangle - \langle u_{2z}\rangle = -\frac{D_{12}}{y_1 y_2}\left(\frac{d}{dz}y_1 + y_1 y_2 \alpha_p p_0^{-1}\frac{dp}{dz} + y_1 y_2 \alpha_T T_0^{-1}\frac{dT}{dz}\right)$$

The coefficients of diffusion, baro- and thermodiffusion are then linked to respective coefficients Λ_{ik} by the relations

$$D_{12} = \Lambda_{11} y_1 y_2 p_0 T_0^{-1}, \quad D_{12}\alpha_p = \Lambda_{1m} p_0 T_0^{-1}, \quad D_{12}\alpha_T = \Lambda_{1q} T_0^{-1}$$

Note that in (3.4) and (3.5) the terms Λ_{ik} containing d^{-1} (d is the channel width) are of the order of the Knudsen number. Coefficients D_{12} and α_T coincide with $[D_{12}]_2$ and $[\alpha_T]_1$ as $\mathrm{Kn}\to 0$. Expression for the constant of barodiffusion α_p differs from parameter $[\alpha_p]_2$ as $\mathrm{Kn}\to 0$ from that in /5/ by the additional term dependent on the nature of scattering of molecules on the wall. When $\varkappa_1 = \varkappa_2 = 1$ (fully diffused scattering) the formula for α_p may be reduced to the form /14/

$$\alpha_p = {}^1/_2\,([\alpha_p]_2 + \alpha_p{}^k)$$

$$\alpha_p{}^k = \frac{(\sqrt{m_2} - \sqrt{m_1}) - 0.5\,(a_1\sqrt{m_1}y_2^{-1} + a_2\sqrt{m_2}y_1^{-1})}{\sqrt{m_1}y_1 + \sqrt{m_2}y_2}$$

To illustrate the dependence of $D_{12}, \alpha_p, \alpha_T$ on the properties of molecules, the Knudsen number, and the nature of molecule scatter on the wall we consider a mixture with a small relative difference of masses and diameters of scatter of component molecules $((m_2 - m_1)/(m_2 + m_1) \ll 1$ and $(\sigma_2 - \sigma_1)/(\sigma_2 + \sigma_1) \ll 1)$ using solid balls of diameters σ_1, σ_2, respectively, as models of molecules. Taking also into consideration the possibility of small difference in the coefficients of molecule reflection on the wall, after simplification of respective formulas (for a mixture with $y_1 = y_2 = 0.5$), we obtain

$$D_{12} = [D_{12}]_2(1 - 0.6523\varkappa\,\mathrm{Kn})$$

$$D_{12}\alpha_p = [D_{12}]_2\Big\{[1.1441 + \varkappa\,(0.1303 - 1.3152\,\mathrm{Kn})]\frac{\Delta m}{2m} + [0.0678 - \varkappa\,(0.6653 - 1.6794\,\mathrm{Kn})]\frac{\Delta\sigma}{2\sigma} + [1.9322 - \varkappa\,(0.0339 - 0.1785\,\mathrm{Kn})]\frac{\Delta\varkappa}{2\varkappa}\Big\}$$

$$D_{12}\alpha_T = -[D_{12}]_2\Big[(0.8898 - 1.1114\,\varkappa\,\mathrm{Kn})\frac{\Delta m}{2m} + (0.3390 - 0.3086\varkappa\,\mathrm{Kn})\frac{\Delta\sigma}{2\sigma} + 0.3443\varkappa\mathrm{Kn}\frac{\Delta\varkappa}{2\varkappa}\Big]$$

$$\frac{\Delta m}{2m} = \frac{m_2 - m_1}{m_2 + m_1}, \quad \frac{\Delta\sigma}{2\sigma} = \frac{\sigma_2 - \sigma_1}{\sigma_2 + \sigma_1}, \quad \frac{\Delta\varkappa}{2\varkappa} = \frac{\varkappa_2 - \varkappa_1}{\varkappa_2 + \varkappa_1}$$

$$\mathrm{Kn} = \frac{5\sqrt{\pi}}{8}\,(\sqrt{2}\,\pi n_0 \sigma^2 d)^{-1}$$

As previously indicated $D_{12}\to[D_{12}]_2$, $\alpha_T\to[\alpha_T]_1$ as $\mathrm{Kn}\to 0$. As regards α_p when $\mathrm{Kn}\to 0$ then for $\varkappa = 1$ we have /14/

$$\alpha_p = 1.2744\,\frac{\Delta m}{2m} - 0.5975\,\frac{\Delta\sigma}{2\sigma}$$

Note that in the formula for $[\alpha_p]_2$ in /5/ the respective coefficients are 1.405 and 1.263. Thus the allowance for Knudsen layers in the channel in the calculation of α_p as $\mathrm{Kn}\to 0$ yields a twice weaker dependence on the relative difference in transverse collisions of molecules, and virtually little changing dependence on the difference of mass of component molecules.

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